

CONTROL OF INTEGRATING SYSTEMS USING A TWO-DEGREE-OF-FREEDOM PID CONTROLLER

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ABSTRACT

This article deals with the comparison of a two-degree-of-freedom (2DOF) PID controller and the classical 1DOF PID controller for the control of integrating systems. The work responds to the structural limitation of the 1DOF controller, which cannot simultaneously optimize setpoint tracking and disturbance rejection. Three typical integrating plants (with transport delay, oscillatory, non-oscillatory) and four controller variants tuned by the SIMC method and the PID Tuner application were tested in the MATLAB Simulink environment. Control quality was assessed using the IAE and ITAE criteria. The serial 2DOF controller improved ITAE by 28 to 54% compared with the best 1DOF solution; the optimal tuning method, however, depends on the plant structure.

Keywords: integrating systems, 2DOF PID, SIMC, PID Tuner, IAE, ITAE

JEL Code: C88

1 INTRODUCTION

Systems with integrating behaviour belong among the more difficult control tasks. Unlike proportional plants they have no self-regulating capability: once disturbed from equilibrium, the output keeps rising or falling until the controller intervenes. Typical examples include the liquid level in a tank without outflow, motor position control or temperature regulation in a greenhouse, where heat losses fully determine the energy balance.

Industrial practice relies on the PID controller with one degree of freedom (PID 1DOF), which takes the control error $e=w-y$ as the only input. This structure has one inherent limitation: tuning the controller for fast response to a setpoint change yields a high gain and consequently high sensitivity to disturbances — and vice versa. Åström and Hägglund (1995) labelled this trade-off as the fundamental limitation of single-input controllers. The aim of the present work is to verify experimentally whether PID 2DOF removes this trade-off and, if so, which tuning method should be selected (Sladká 2020).



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1.1 PID 2DOF and its advantages

PID 2DOF has structurally two parallel paths: an input filter for the setpoint w with weights b and c for the proportional and derivative term, and a classical PID channel for the control error e (Šulc and Vítečková 2004; Wang 2020). The serial variant proposed by Vítečková and Víteček (2011) introduces the discrete transfer functions of the input filter $G_F(z)$ and of the controller $G_R(z)$:

$$G_F(z) = \frac{A_g^{(1)} z^2 - B_g^{(1)} z + C_g^{(1)}}{A_g z^2 - B_g z + C_g}, \quad G_R(z) = \frac{K_p(A_g z^2 - B_g z + C_g)}{Z_2 z^2 - Z_1 z + Z_0} \quad (1)$$

This structure allows the tracking mode (through the filter) and the disturbance-rejection mode (through the controller) to be tuned independently. The dissertation by Sladká (2020) demonstrated the applicability of 2DOF on typical industrial processes; the open question remained the behaviour on integrating plants, where long-term disturbance rejection is usually the dominant task.

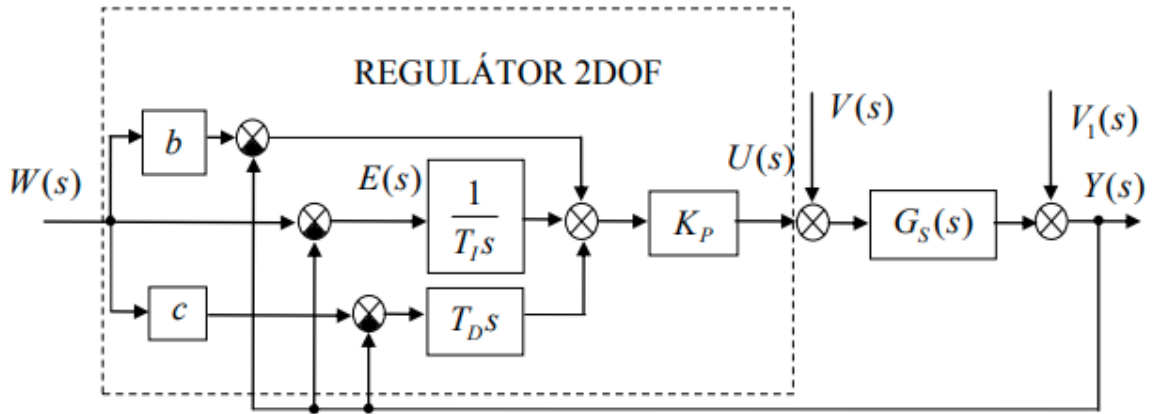


Fig. 1: Block diagram of the control loop with an analog PID 2DOF controller. The set-point $W(s)$ is weighted by the coefficients b (proportional) and c (derivative), the control error $E(s)$ enters the integral path, and the resulting actuating signal $U(s)$ drives the plant $G_S(s)$ with additive disturbance $V(s)$ (adopted from Meduna 2024).

1.2 Tuning methods

Hundreds of empirical rules exist for 1DOF controllers (O'Dwyer 2009). The present work compares two methods used in practice. The SIMC (Simple Internal Model Control) method derives analytical expressions for the parameters K_p , T_p , T_D from the internal model of the plant with a single tuning parameter T_w (Víteček and Vítečková 2011). PID Tuner from the MATLAB Control System Toolbox searches the parameter space automatically; for the 2DOF case it adds parameters T_o (time response) and T_b (robustness) (Mathworks 2024).

2 METHODOLOGY

Simulation circuits were built in MATLAB Simulink with a sampling period of $T=0.1$ s for three typical integrating plants.

Integrating plant with transport delay ($T_s = 1 \cdot \text{s}$, $T_1 = 12 \cdot \text{s}$, $T_d = 4 \cdot \text{s}$):

$$G_{s1}(s) = \frac{1}{12s^2 + s} e^{-4s}. \quad (2)$$

In agriculture this corresponds, for example, to material transport that introduces a noticeable delay between actuator action and output response.

Oscillatory integrating plant ($T_s = 1\text{s}$, $T_0 = 5\text{s}$, $\xi = 0.8$) :

$$G_{s2}(s) = \frac{1}{25s^3 + 8s^2 + s}. \quad (3)$$

Non-oscillatory integrating plant ($T_s = 1\text{s}$, $T_1 = 10\text{s}$, $T_2 = 6\text{s}$) :

$$G_{s3}(s) = \frac{1}{60s^3 + 16s^2 + s}. \quad (4)$$

Four controllers were run in parallel for each plant: PID 1DOF (SIMC and PID Tuner) and serial PID 2DOF (SIMC and PID Tuner). A signal shaper acting as a D/A converter followed the controller output. A disturbance $v(t)$ entered the loop in front of the plant as a step change at $t = 200$ s. To test robustness, one time constant was changed at $t = 250$ s (for the transport-delay plant T_1 from 12 s to 14 s), simulating plant ageing.

Control quality was assessed by four metrics: settling time with 1% tolerance, overshoot, and the integral criteria IAE and ITAE:

$$I_{IAE} = \int_0^\infty |e(t)| dt, \quad I_{ITAE} = \int_0^\infty t |e(t)| dt. \quad (4)$$

IAE penalises the absolute magnitude of the control error, ITAE weights later errors more heavily.

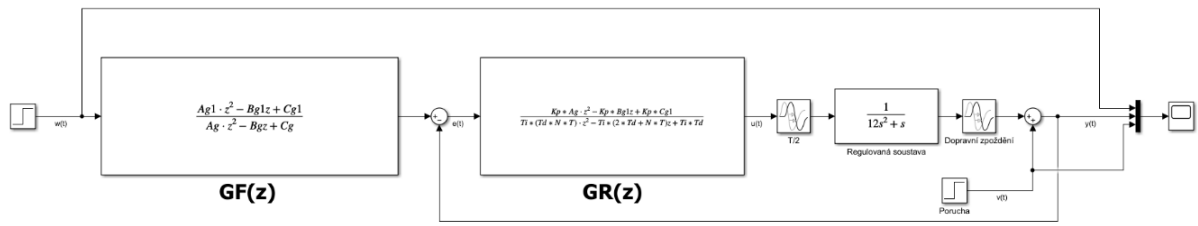


Fig. 2: Schematic of the simulation circuit with a serial PID 2DOF controller realised by transfer functions $G_F(z)$ and $G_R(z)$ in MATLAB Simulink (adopted from Meduna 2024).

3 RESULTS

Tables 1 to 3 summarise the results for a setpoint step at $t = 0$ s; the best variant for each plant is shown in bold.

The serial 2DOF controller won across all three plants. For the transport-delay and non-oscillatory plants the best method was SIMC, for the oscillatory plant PID Tuner. The robustness test under parameter change, however, yielded a different picture for the transport-delay plant: after the change of T_1 from 12 s to 14 s the most stable result came from PID 1DOF tuned by PID Tuner (settling 75.4 s vs 105.8 s for SIMC 2DOF), while the 2DOF controllers lost 50 to 130 % of their original speed. For the oscillatory plant PID Tuner 2DOF held the leading position even after the parameter change (ITAE 180 vs 285 for 1DOF Tuner) (Meduna, 2026).

Tab. 1 Plant with transport delay

Tuning method	Settling [s]	Overshoot [%]	IAE	ITAE[s]
PID Tuner 1DOF	83.42	33.9	14.12	253.11
SIMC 1DOF	94.89	29.4	15.70	315.52
PID Tuner 2DOF	77.38	8.7	17.86	228.27
SIMC 2DOF	45.40	11.9	10.95	147.73

Tab. 2 Oscillatory integrating plant

Tuning method	Settling [s]	Overshoot [%]	IAE	ITAE[s]
PID Tuner 1DOF	96.79	28.2	12.34	242.50
SIMC 1DOF	121.22	21.4	17.38	473.08
PID Tuner 2DOF	64.71	4.7	11.95	111.49
SIMC 2DOF	96.38	7.7	14.68	363.85

Tab. 3 Non-oscillatory integrating plant

Tuning method	Settling [s]	Overshoot [%]	IAE	ITAE[s]
PID Tuner 1DOF	124.61	25.9	16.73	451.93
SIMC 1DOF	140.23	27.0	22.56	699.64
PID Tuner 2DOF	89.79	11.2	20.28	319.73
SIMC 2DOF	62.66	8.4	15.62	324.95

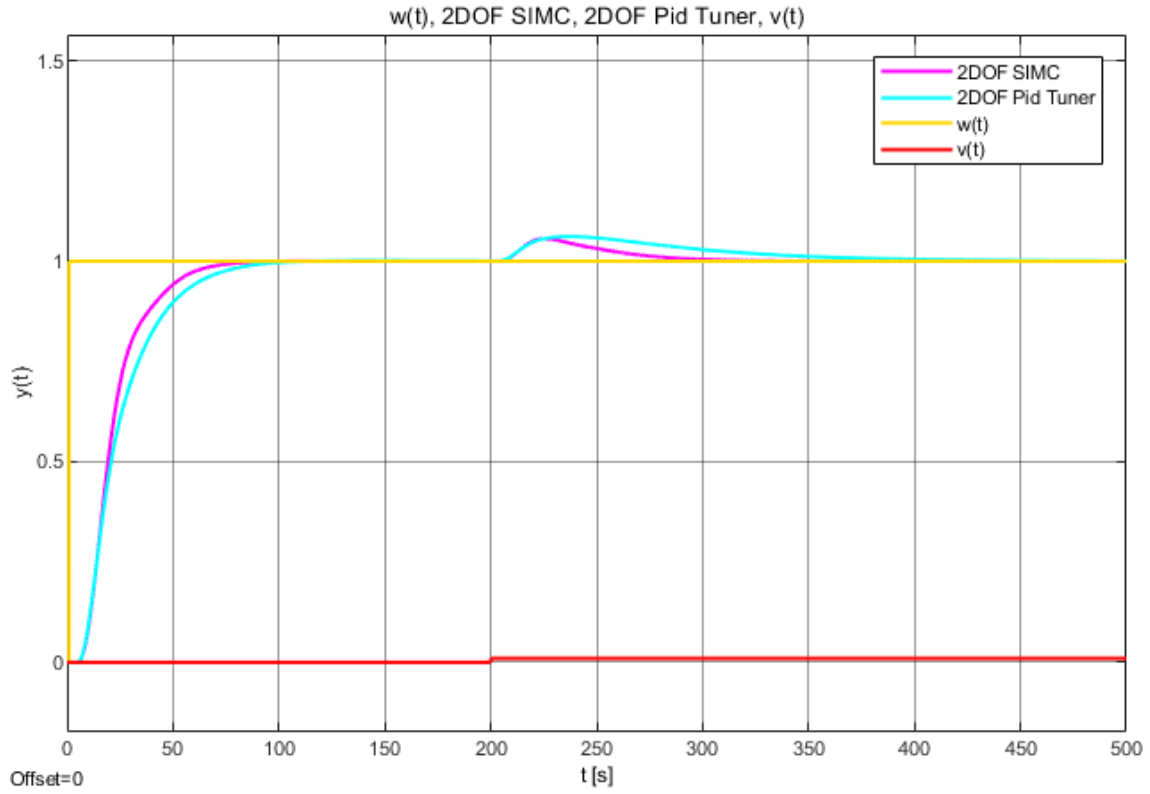


Fig. 3: Control response for the integrating plant with transport delay — comparison of PID 2DOF tuned by SIMC against PID Tuner. The disturbance step $v(t)$ enters at $t = 200$ s (adopted from Meduna 2024).

4 DISCUSSION

The structural advantage of the 2DOF controller was confirmed quantitatively: overshoot dropped from 25–35 % to 5–12% and settling time shortened by 25–50% across all three plants. The reason is the independent tuning of the tracking mode through the input filter G_F and of the disturbance-rejection mode through the controller G_R . A 1DOF controller must choose a higher gain to achieve the same disturbance rejection, which yields an oscillatory response to a setpoint change.

The choice of tuning method depends on plant structure. SIMC produces deterministic, reproducible results whenever an analytical formula exists for the given structure; for the oscillatory integrating plant the formula has to be approximated from a 1st-order template instead of the original 2nd-order one, which explains why PID Tuner outperformed it in this case. The practical rule reads: if SIMC covers the structure, use it as the primary method; otherwise reach for PID Tuner.

Robustness comes at a price. A 2DOF SIMC controller tuned precisely to specific dynamics deteriorated more after a 17% change of T_1 than a less precisely tuned 1DOF Tuner. For processes with substantial model uncertainty (such as agricultural greenhouses with seasonal parameter drift), the more conservatively tuned 1DOF or a 2DOF with an adaptive identification mechanism is preferable.

5 CONCLUSION

The work compared four PID controller variants on three typical integrating plants in MATLAB Simulink. For integrating systems, the serial PID 2DOF is structurally the better choice than the classical PID 1DOF: across all tested plants, it improved ITAE by 28 to 54% and reduced overshoot fivefold. The choice of tuning method, however, depends on the structure — SIMC for transport-delay and non-oscillatory plants, PID Tuner for oscillatory ones. Robustness against parameter changes is lower for 2DOF than for a conservatively tuned 1DOF, which represents the price for precision. Future extensions may include verification of 2DOF on non-linear plants with actuator saturation, adaptive tuning from a continuously identified model, and experimental verification on a physical system such as DC motor position control.

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REFERENCES

- ASTRÖM, K. J., HÄGGLUND, T. 1995. *PID Controllers: Theory, Design, and Tuning*. 2nd ed. Research Triangle Park, NC: Instrument Society of America. ISBN 1-55617-516-7
- MATHWORKS. 2024. *PID Controller Tuner App*. <https://www.mathworks.com/help/control/getstart/tune-pid-controller-to-balance-tracking-and-disturbance-rejection.html>
- MEDUNA, J. 2026. *Control of integrating systems using a two-degree-of-freedom PID controller* [in Czech]. Master's thesis, head: Farana, R. Brno: Mendel University in Brno.
- O'DWYER, A. 2009. *Handbook of PI and PID Controller Tuning Rules*. 3rd ed. London: Imperial College Press. ISBN 978-1-84816-242-6
- SLADKÁ, K. 2020. *Tuning of Digital and Analog Two-Degree-of-Freedom Controllers* [in Czech]. PhD Dissertation. Ostrava: VŠB – Technical University of Ostrava, Faculty of Mechanical Engineering.
- ŠULC, B., VÍTEČKOVÁ, M. 2004. *Theory and Practice of Control Loop Design* [in Czech]. Prague: Czech Technical University Publishing House. ISBN 80-01-03007-5
- VÍTEČEK, A., VÍTEČKOVÁ, M. 2011. *Selected Methods of Controller Tuning* [in Czech]. Ostrava: VŠB – Technical University of Ostrava. ISBN 978-80-248-2503-8
- WANG, L. 2020. *PID Control System Design and Automatic Tuning using MATLAB/Simulink*. London: John Wiley & Sons. ISBN 978-1-119-46934-6

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