THE USABILITY OF ALGORITHMS FROM GRAPH THEORY IN THE FIELD OF MULTICRITERIA ANALYSIS

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ABSTRACT

The main ideas, on how to use algorithms from the Graph Theory to improve the process of Multicriteria Analysis were presented in the paper (Farana, 2016). This paper presents practical results obtained by the use of these algorithms in practical decision-making procedures when multicriteria analysis has been used.

Graph algorithms were used in two situations when determining the values of the weights of decision criteria. First was the checking, if the Fuller's triangle, filled in by an expert, is filled in correctly. For this verification, a complete graph is used in which the vertices represent the criteria the orientation of the edges their mutual significance. A method of gluing vertices could be used for criteria with the same significance. The resulting graph must be acyclic. Twenty-five decision tasks with seven or more criteria were analyzed and the obtained results will be presented in the paper.

The second application was the elimination of the overdetermination of the assessment in Saaty's method. A spanning tree describing dependencies between criteria has been used according to the algorithm in (Farana, 2016). Obtained results were compared with the full Saaty's matrix when the number of compared pairs of criteria is k – 1 for k criteria, compared to the number of k(k – 1)/2 in the classic Saaty's method. Fifteen decision tasks with seven or more criteria were analyzed and the obtained results will be presented in the paper. The paper presents the differences between the assessment given directly by experts and the assessment obtained using the spanning tree and shows that the described method is applicable in practice. The experience of experts using the proposed procedure, obtained through a guided interview, was mostly positive.

Keywords: decision making, multicriteria analysis, graph theory, Fuller triangle, Saaty's method, acyclic graph, spanning tree

JEL Code: D81 Criteria for Decision-Making under Risk and Uncertainty

https://doi.org/10.11118/978-80-7509-990-7-0060

1 INTRODUCTION

Methods of multi-criteria analysis of variants (multi-criteria decision-making) are described in several publications, e.g. (Multi-criteria analysis, 2009; Triantaphyllou, 2000; Jablonský, 2007; Malakooti, 2013) and are still the source of several applications in solving complicated decision-making tasks, see e.g. (Borovcová, 2010). There are also many publications focused on the quality of value estimation and comparison of different approaches, see e.g. (Agarski, 2019).

Number of criteria	Number of experts	Expert experience	Number of problems	Longer cycle length
7	7	Experienced	0	0
9	8	Very experienced	0	0
8	6	Inexperienced	1	3
11	10	Experienced	1	3
7	8	Inexperienced	0	0
9	7	Inexperienced	0	0
8	5	Experienced	0	0
8	7	Very experienced	0	0
9	7	Inexperienced	1	3
10	12	Experienced	1	4
12	11	Experienced	0	0
7	7	Very experienced	0	0
8	7	Very experienced	0	0
7	9	Inexperienced	1	3
9	7	Experienced	0	0
9	9	Experienced	0	0
10	8	Inexperienced	0	0
8	6	Very experienced	0	0
7	7	Experienced	0	0
8	5	Experienced	0	0
9	7	Very experienced	0	0
12	8	Experienced	2	3
8	7	Experienced	0	0
7	5	Very experienced	0	0
9	7	Inexperienced	0	0

Tab. 1 Results of using Fuller's triangle in implemented multi-criteria decision-making tasks

We use the knowledge of experts to choose the best option while respecting a large number of, often conflicting, criteria. Above all, in the phase of determining the importance of individual criteria. Several methods have been gradually developed that try to help experts in determining the importance of criteria, especially when there are more of them. The Fuller's triangle and the Saaty's method are particularly effective methods. Unfortunately, the facilitation of expert decision-making is balanced by the danger of conflicting evaluation, which can negatively affect the result of the entire multi-criteria analysis, because we know from practice that these tasks are often very sensitive to the significance of the criteria. In other words, even a small change in the significance of the criteria can cause a significant change in the resulting evaluation of the variants.

The paper (Farana, 2016) presented how we can effectively use methods known from Graph Theory, e.g. (Gross, 2006), to detect these inconsistencies or even avoid them. This paper presents practical experiences from the use of these methods.

2 FULLER'S TRIANGLE

Fuller's triangle (also called the Pairwise comparison method), is a way to compare and determine the significance of a large number of evaluation criteria. The expert is presented with a set of all pairs of criteria with a request to mark which of the pair is more important, or may also mark both as equally important. This greatly simplifies his decision-making on the one hand, but at the same time, there is a danger that his opinion will be inconsistent.

This contradiction cannot be resolved and in more complex cases it is possible that it will not even be detected. For this, we can advantageously use a graph (complete), in which the vertices represent the criteria and the edges of their mutual evaluation. Firstly, we use the method of gluing for vertices with the same importance, next we orient the edges towards a more significant criterion. The resulting graph must be acyclic. If the graph contains cycles, the evaluation is inconsistent.

Table 1 presents the obtained results of using Fuller's triangle in implemented multi-criteria decision-making tasks with 7 or more criteria. It is evident that a very small number of problems have been identified. The typical length of the cycle is 3. Experts with little experience made the most mistakes.

A structured interview method was used to identify the source of faults. Most often, the experts stated that they found the criteria to be very similarly important, but for some reason, they did not mark them as equally important. This also shows the possibility of removing the cycle by marking all criteria in the cycle as equally important.

An interesting fact was discovered during the discussion with very experienced experts. A number of them stated that they were aware of the danger of inconsistent evaluation. And they prevent it by ranking the criteria in order of importance before they begin to fill in Fuller's triangle. Then the question is whether to use directly the method of order to determine the significance of the criteria.

3 SAATY'S METHOD

Saaty's method is a well-known method that enables a more sensitive evaluation of criteria (Saaty, 1977). Expert is evaluating every pair of criteria using values from 1 to 9 to determine the strength of preferences. Since all pairs of criteria are compared, the same problem as in Fuller's triangle can arise, i.e. the creation of a cycle. Then also the same method to identify the cycle, described above, can be used. However, the possibility to express the importance of preferences brings additional risks of inconsistency in evaluation. To eliminate overdetermination of evaluations, it is possible to use a graphic interpretation of the relationships

	A	B	C	D	E
A	1	1	1/5	1/3	5
B	1	1	1/5	1/3	5
C	5	5	1	3	9
D	3	3	1/3	1	7
E	1/5	1/5	1/9	1/7	1

Tab. 2 Saaty's table calculated from the expert's evaluations

between the criteria in the form of a graph. In this graph, we will find the spanning tree (any of them), we will evaluate criteria pairs at the spanning tree only, and calculate the rest of the evaluations (Farana, 2016).

Table 2 presents an example of five criteria (A to E) evaluation when only pairs A-B, B-C, A-D, and A-E were given by an expert and all rest values were calculated. It is evident, that the main diagonal contains values of 1, and the reciprocal values we obtain according to the relation (B-A) = 1/(A-B).

The next procedure already respects Saaty's method, so we determine the weights of the criteria and we can check the consistency of Saaty's matrix by calculating the variance estimate. The resulting value for the example from Table 2 is 0.155, and for five criteria a variance estimate of less than 0.2 is required.

Fifteen decision tasks with seven or more criteria were analyzed and experts were asked about their opinion on this method. The obtained results were significantly dependent on the experience of the experts:

Very experienced – neutral opinion, they do not see a problem in the overdetermination of the assessment, and they are not concerned about inconsistent assessment.

Experienced – predominantly positive opinion, they see the danger in the overdetermination of the assessment, and they are concerned about inconsistent assessment.

Inexperienced – a completely positive opinion, they see a danger in the overdetermination of the assessment, and they are very concerned about inconsistent assessment. They prefer to provide as few ratings as possible.

4 RESULTS

The paper presented two possible applications of algorithms from graph theory using classical methods for determining the significance of criteria. Sophisticated methods of evaluating the importance of criteria such as Fuller's triangle or Saaty's method carry the risk of inconsistent evaluation. The presented applications of algorithms from graph theory allow inconsistencies to be detected or even eliminated.

5 DISCUSSION AND CONCLUSIONS

The use of graph algorithms gives new options to support decision-making, many publications are focused on this area, see e.g. (Nordeman 2020, Nestrenko 2022). But classical methods are still used and the use of presented procedures and applications from graph theory can significantly support them. The results achieved and the opinions of experts, presented in this paper, prove it.

Acknowledgements
This paper was supported by the project CZ.02.1.01/0.0/0.0/16_017/0002334 Research Infrastructure for Young Scientists, this is co-financed by Operational Programme Research, Development and Education.

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